# Tightly-Secure Blind Signatures in Pairing-Free Groups


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|---|---|
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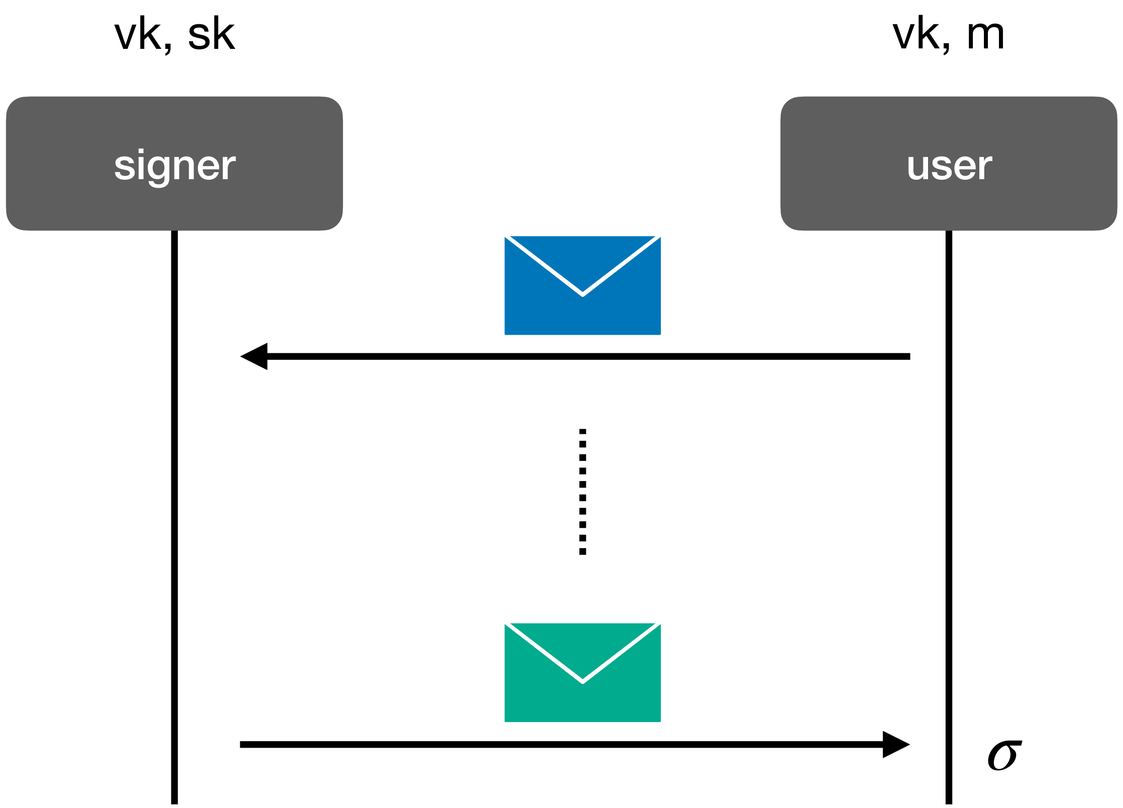
| Michael Reichle | ETH Zurich |

# Blind Signatures

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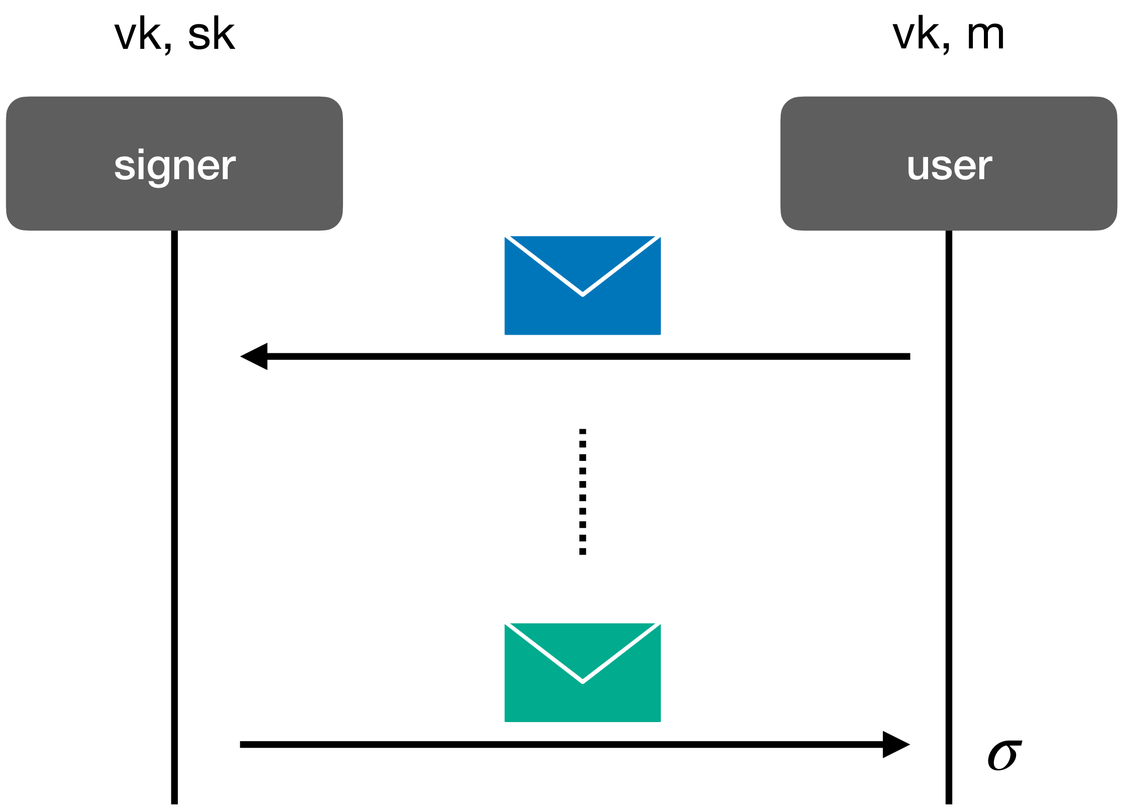


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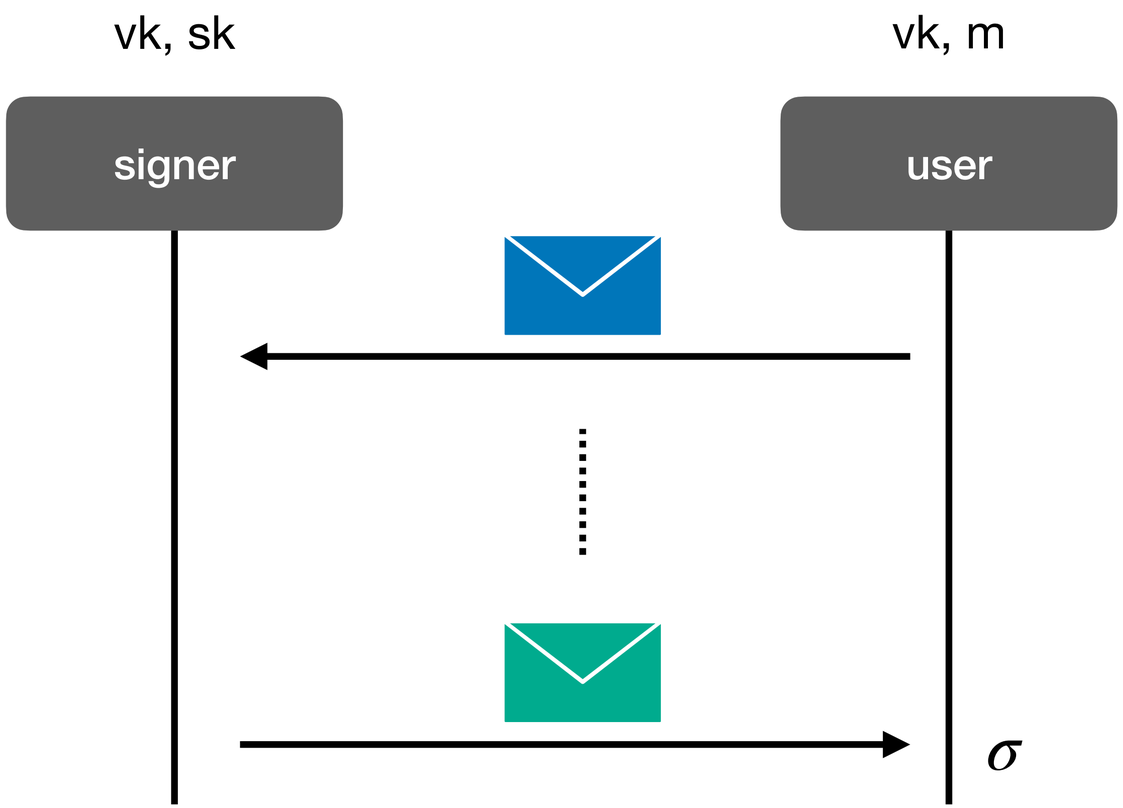


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One-more Unforgeability:

- user can obtain at most $Q_S$ signatures from $Q_S$ sessions with distinct messages

# Our Contribution

- ***Tight blind signatures in pairing-free groups***
  - secure under DDH with $\log(Q_S)$ loss
  - *extension:* non-programmable ROM
  - *extension:* predicate blindness

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## Blind Signatures in Pairing-free Groups

| Scheme | Signature Size | Communication Size | Security | Assumption | Loss |
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| **[CKMTZ23]** | $3(\mathbb{G}/\mathbb{Z}_p)$ | $6(\mathbb{G}/\mathbb{Z}_p)$ | AGM + ROM | DL | $O(Q_S)$ |
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| **Our Work** | $41(\mathbb{G}/\mathbb{Z}_p)$ | $83(\mathbb{G}/\mathbb{Z}_p)$ | ROM | DDH | $O(\log Q_S)$ |

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  - **result:** pairing-free blind signature without AGM

❌ loss is $\Omega(Q)$

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    $\rightarrow$ compile via Fiat-Shamir
  - argue security via adaptive partitioning **[Hof17,AHNOP17]**

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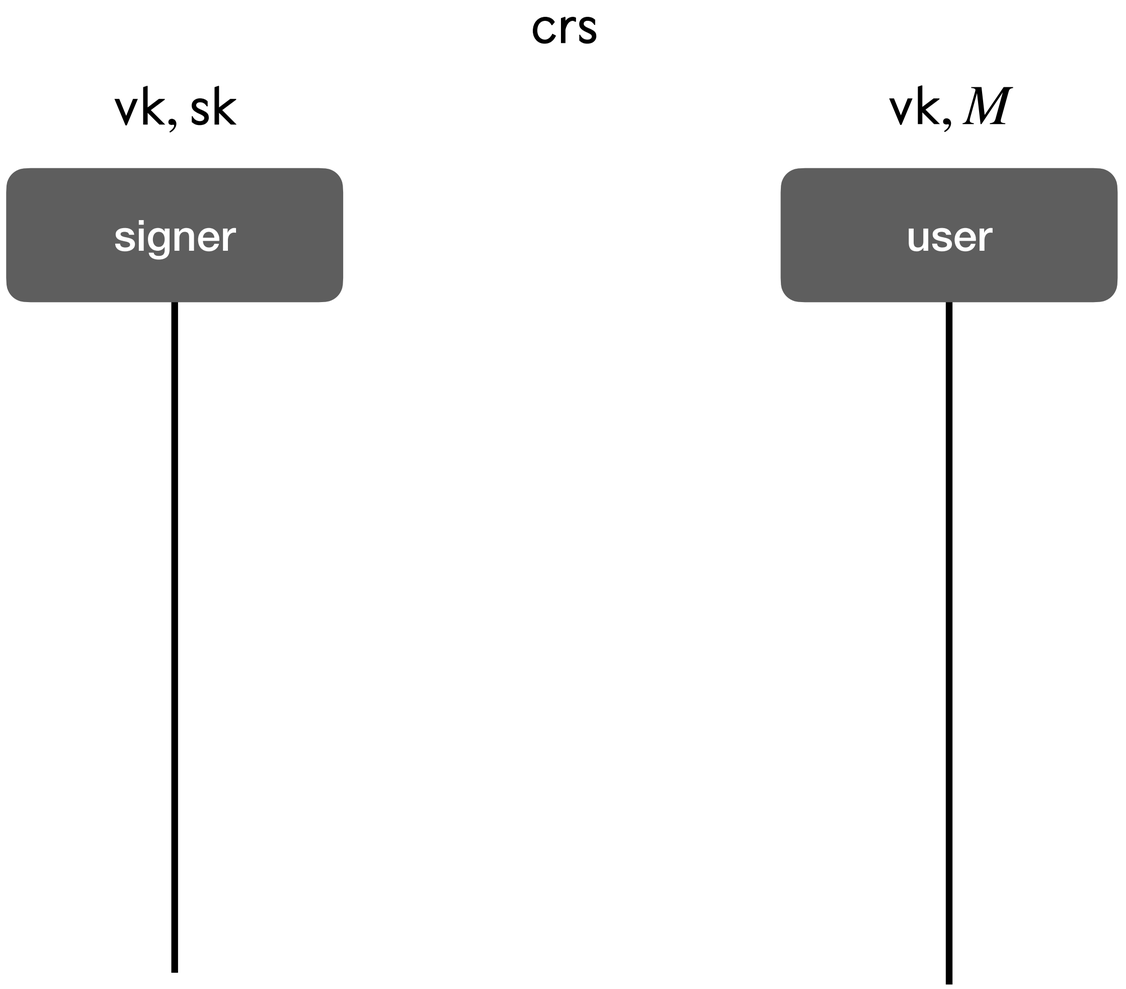

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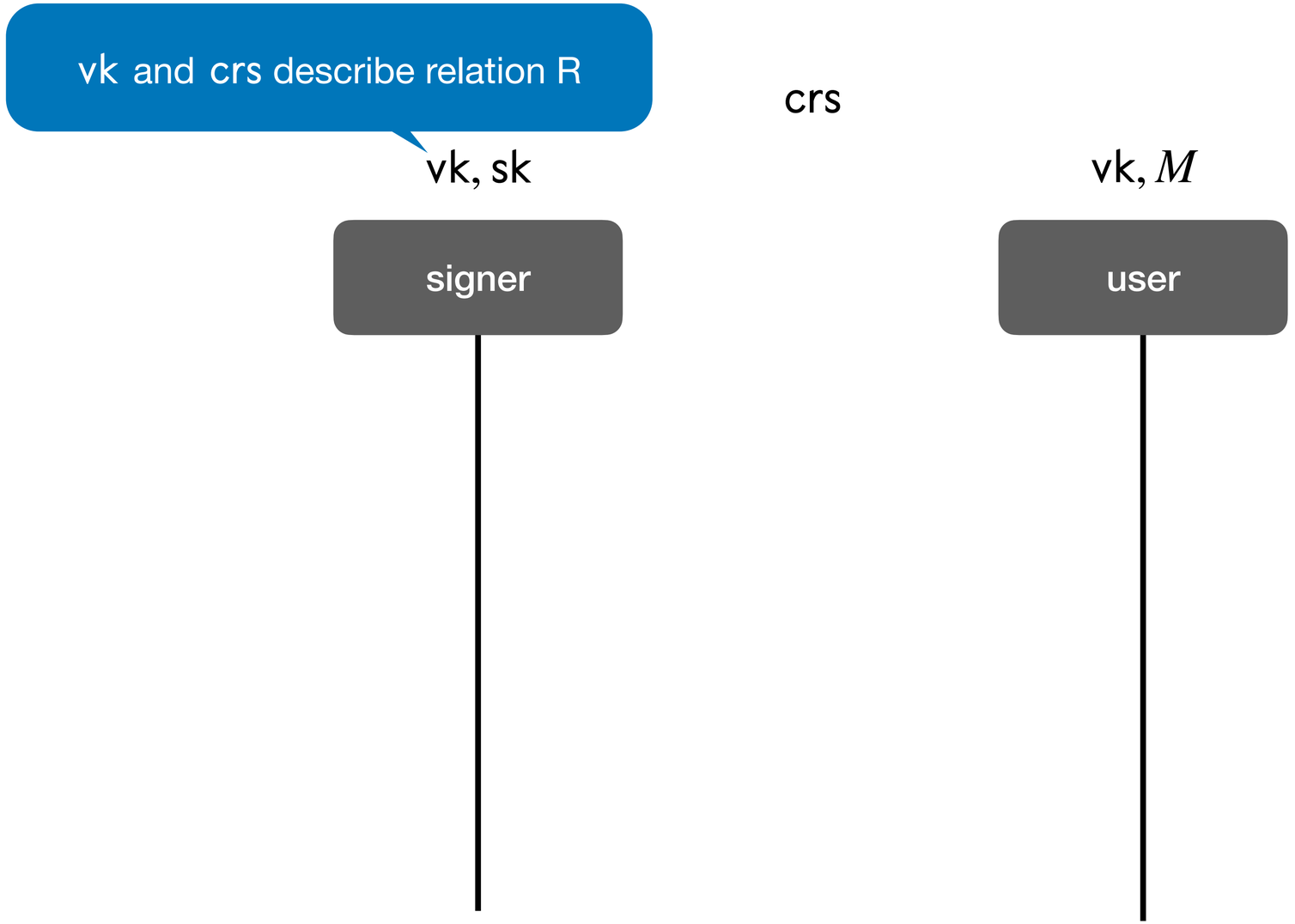

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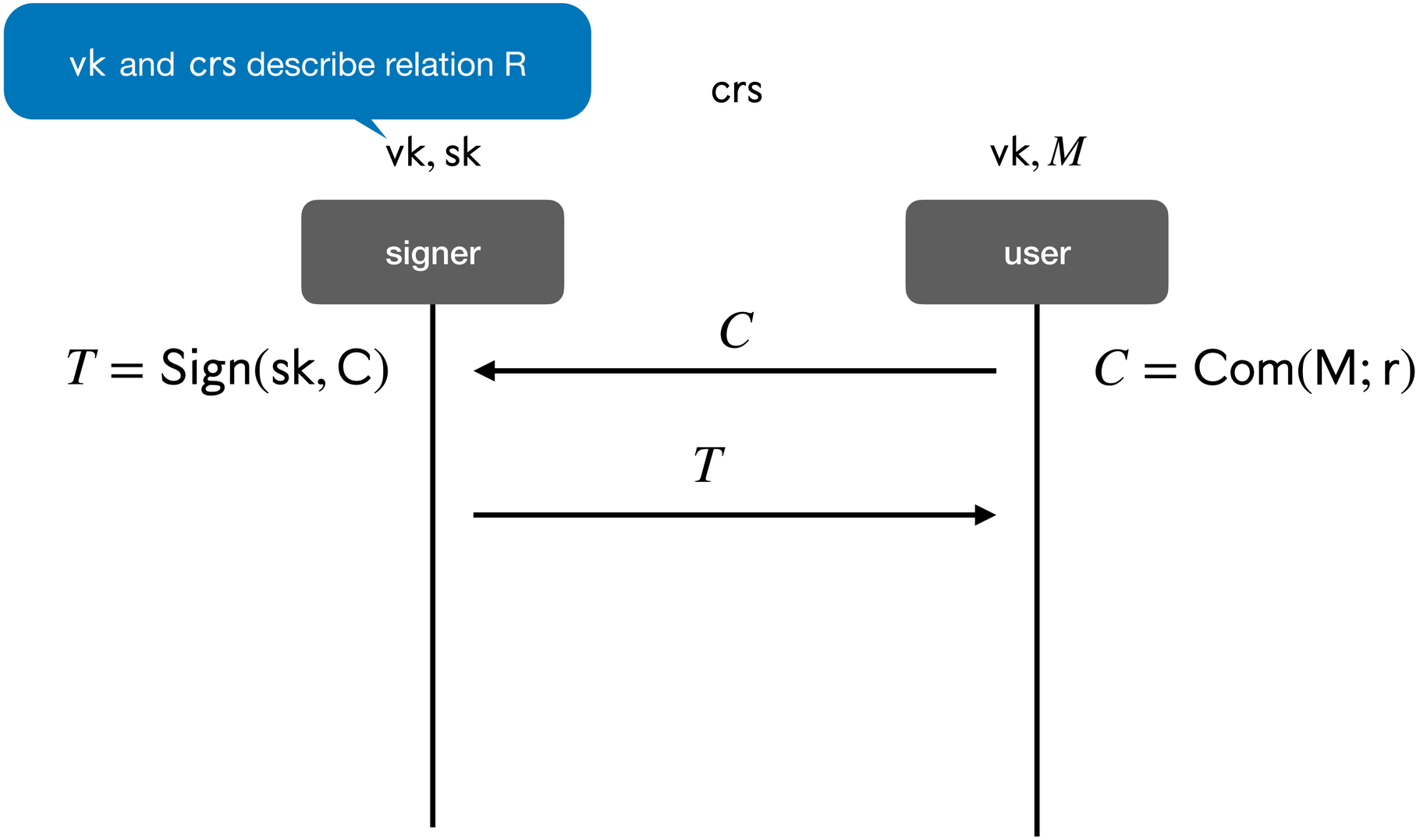

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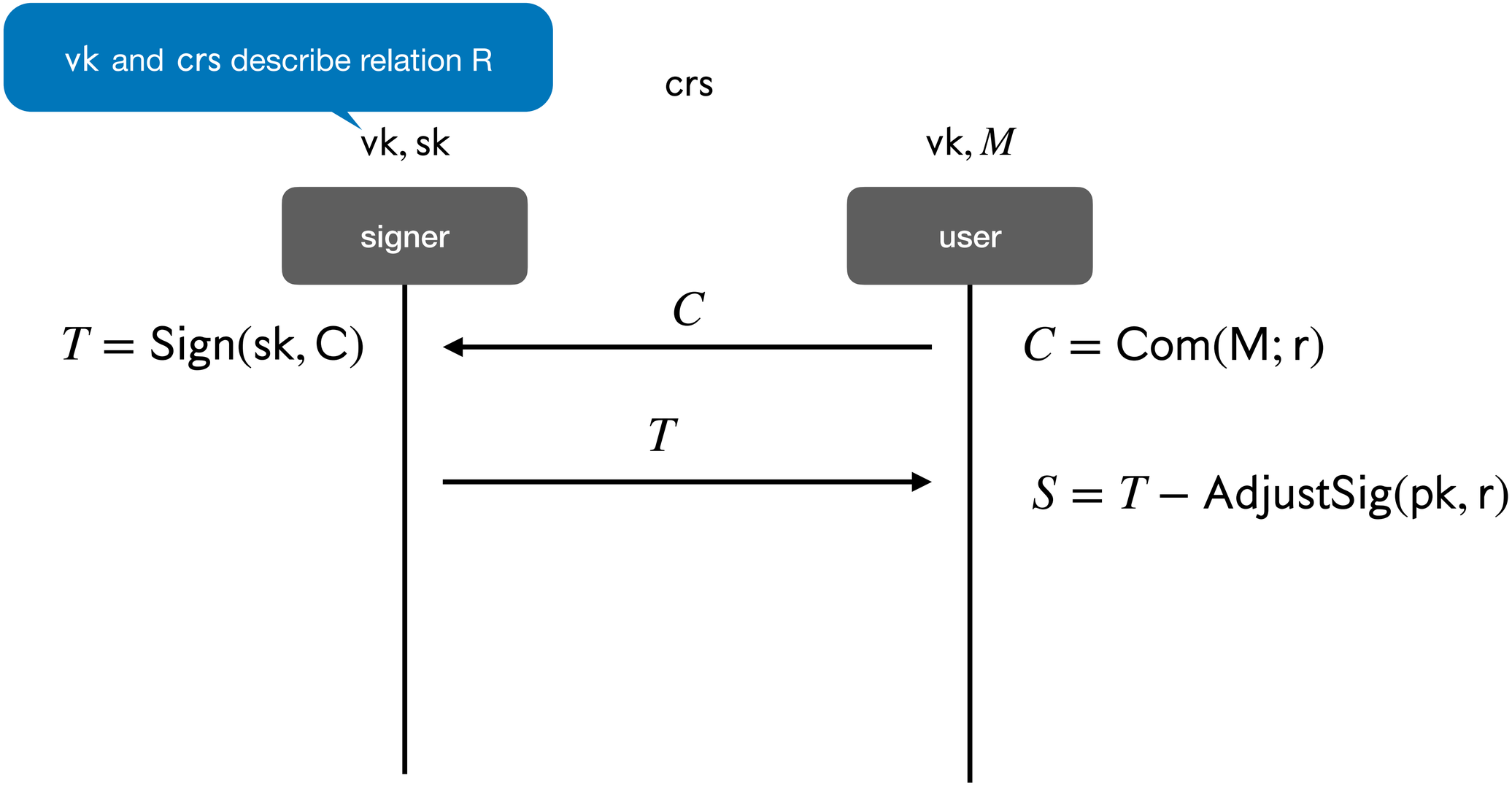

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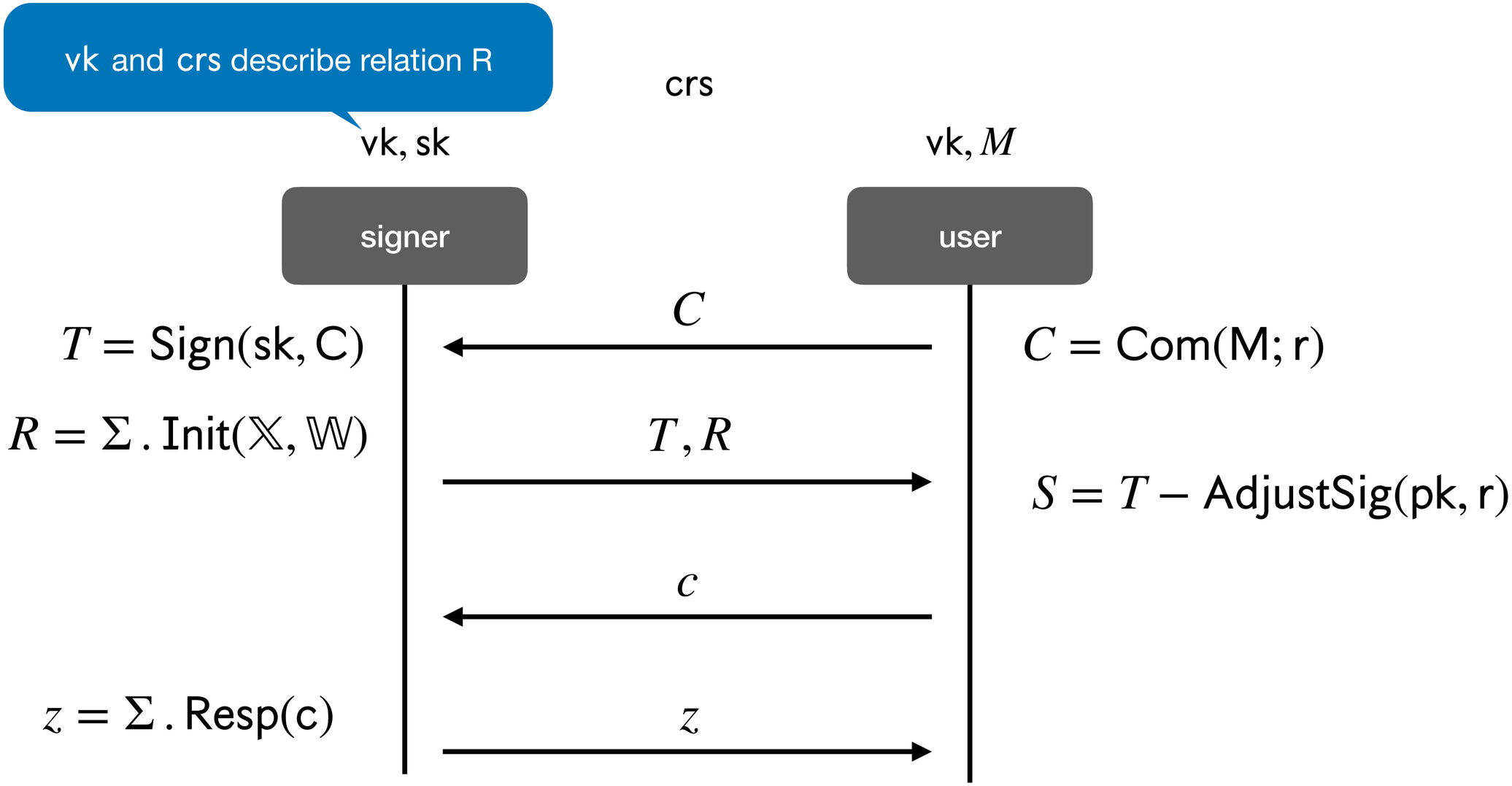

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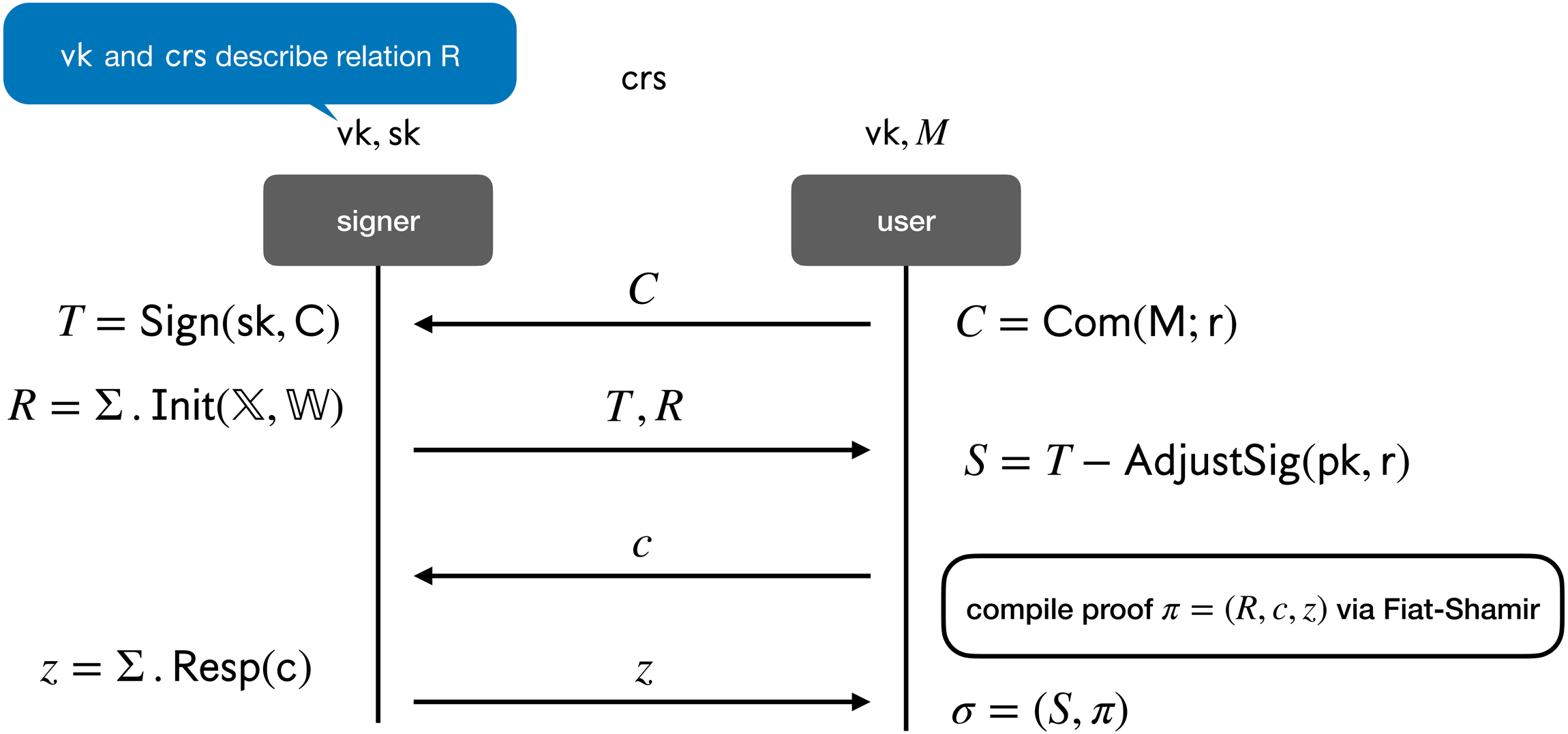

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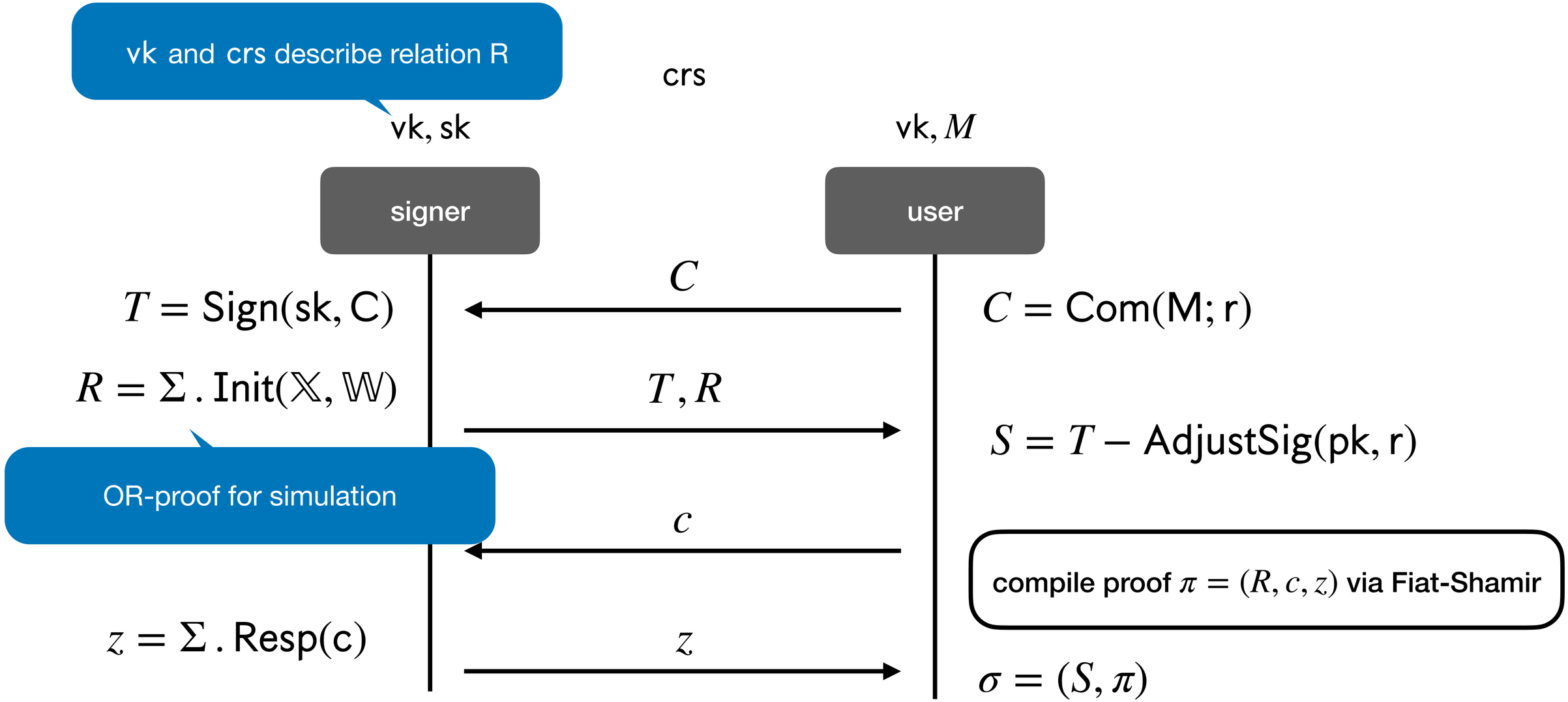

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- $\sigma = (\mathsf{CT}, \pi)$
  - $\mathsf{CT} = (\mathsf{CT}_0, \mathsf{CT}_1, \mathsf{CT}_2)$ // encryptions of $Z_i$
  - $\pi = (\pi_0, \pi_1)$ for relations $R_0, R_1$ // compiled via Fiat-Shamir

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**issue:** statement contains M

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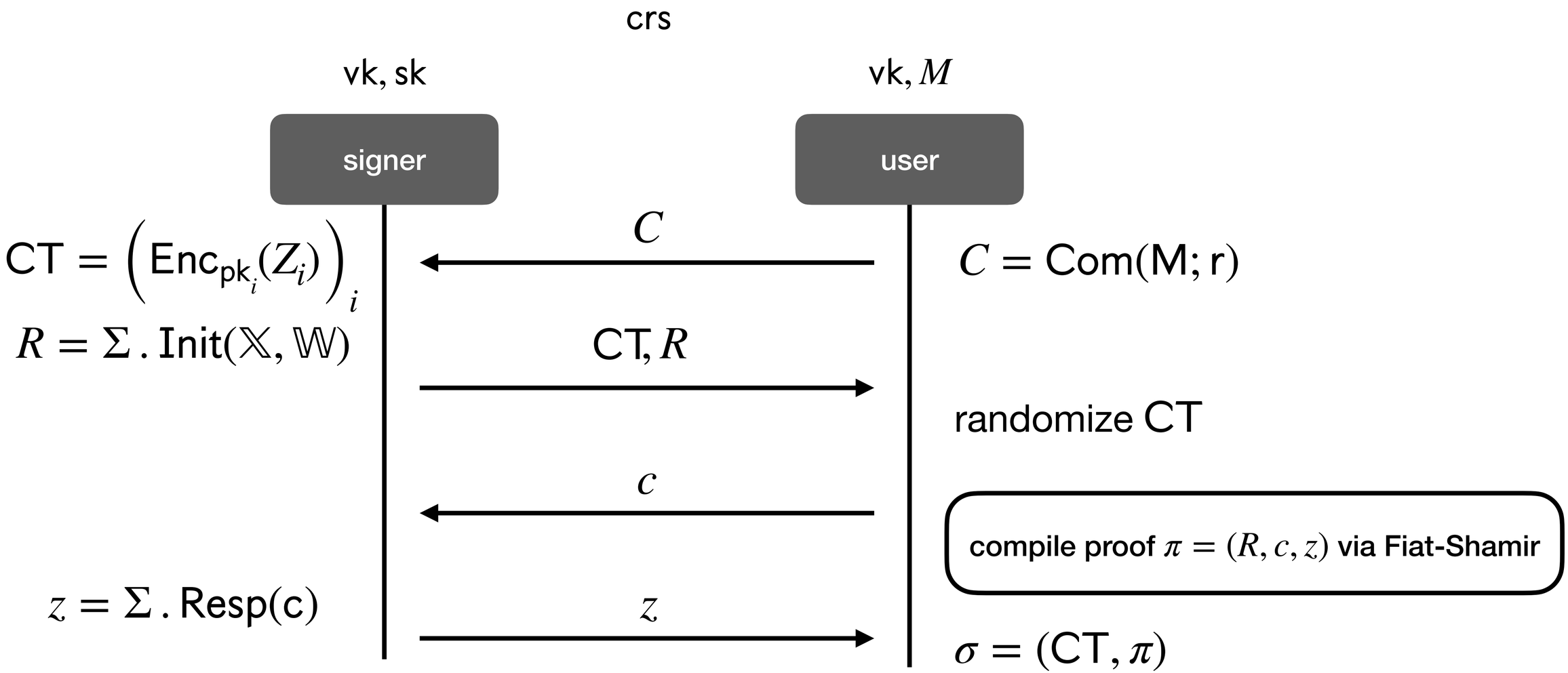

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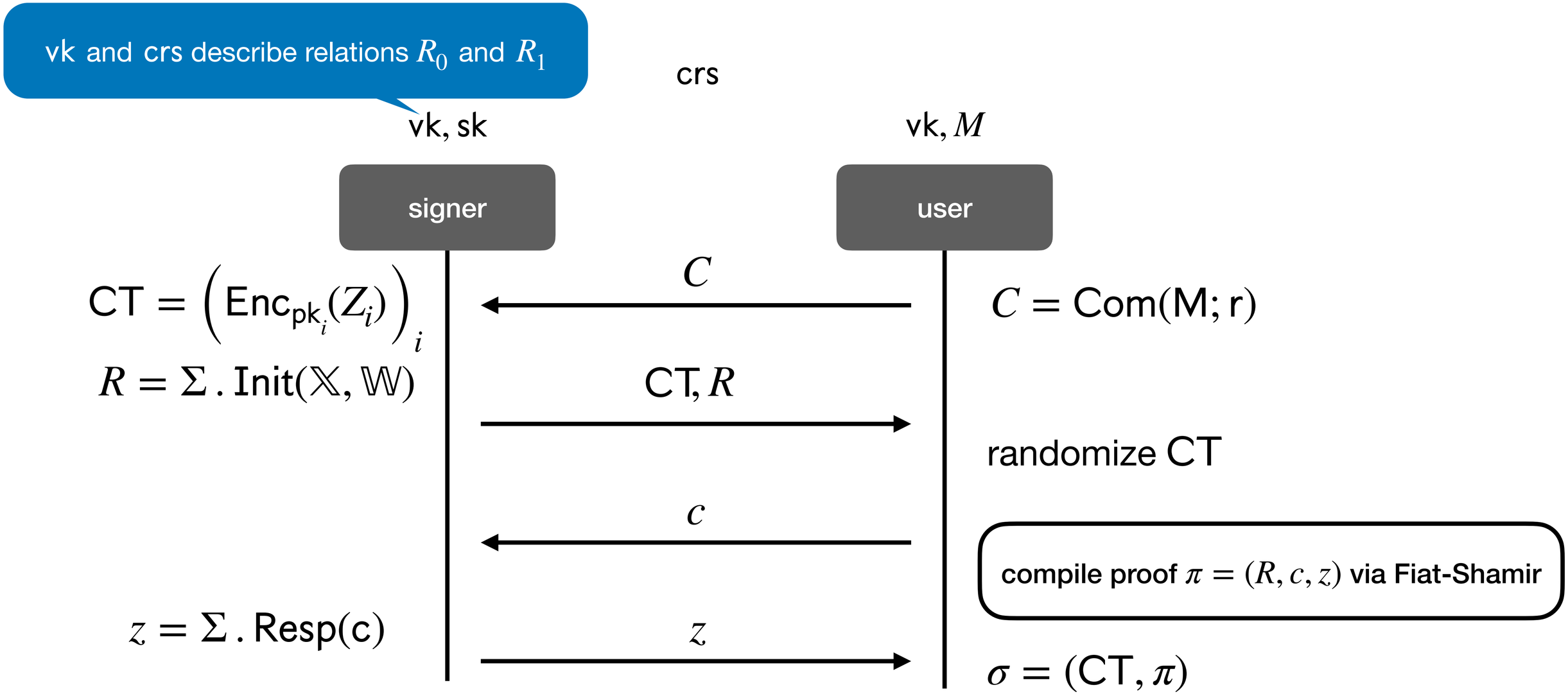

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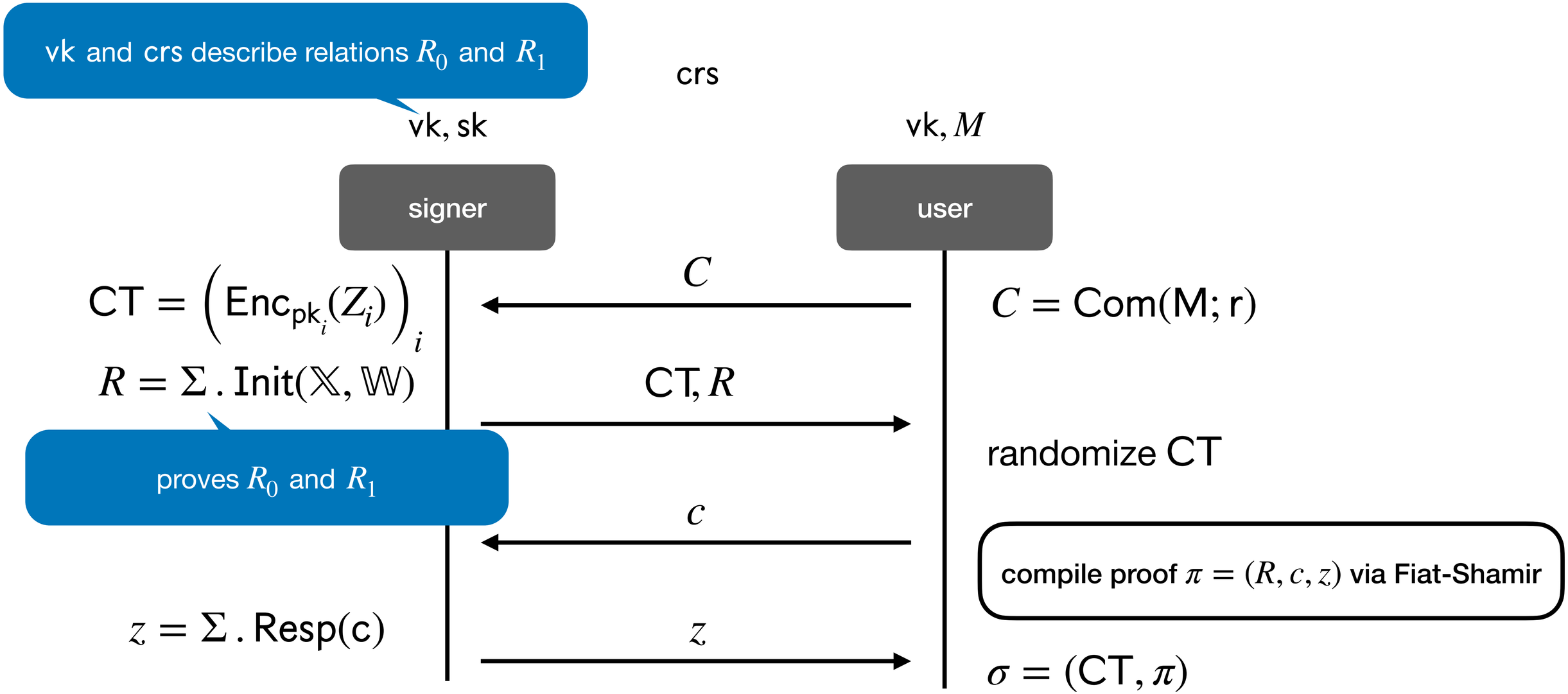

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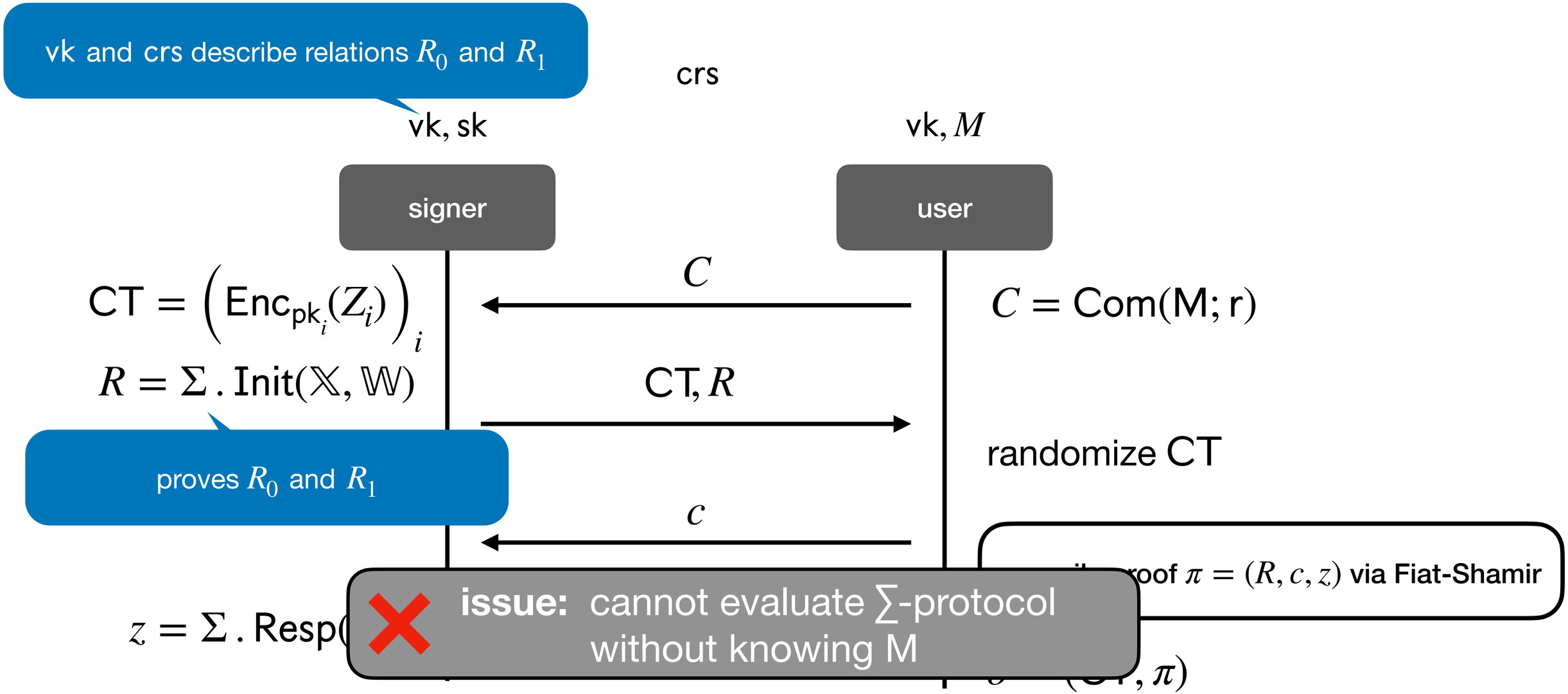

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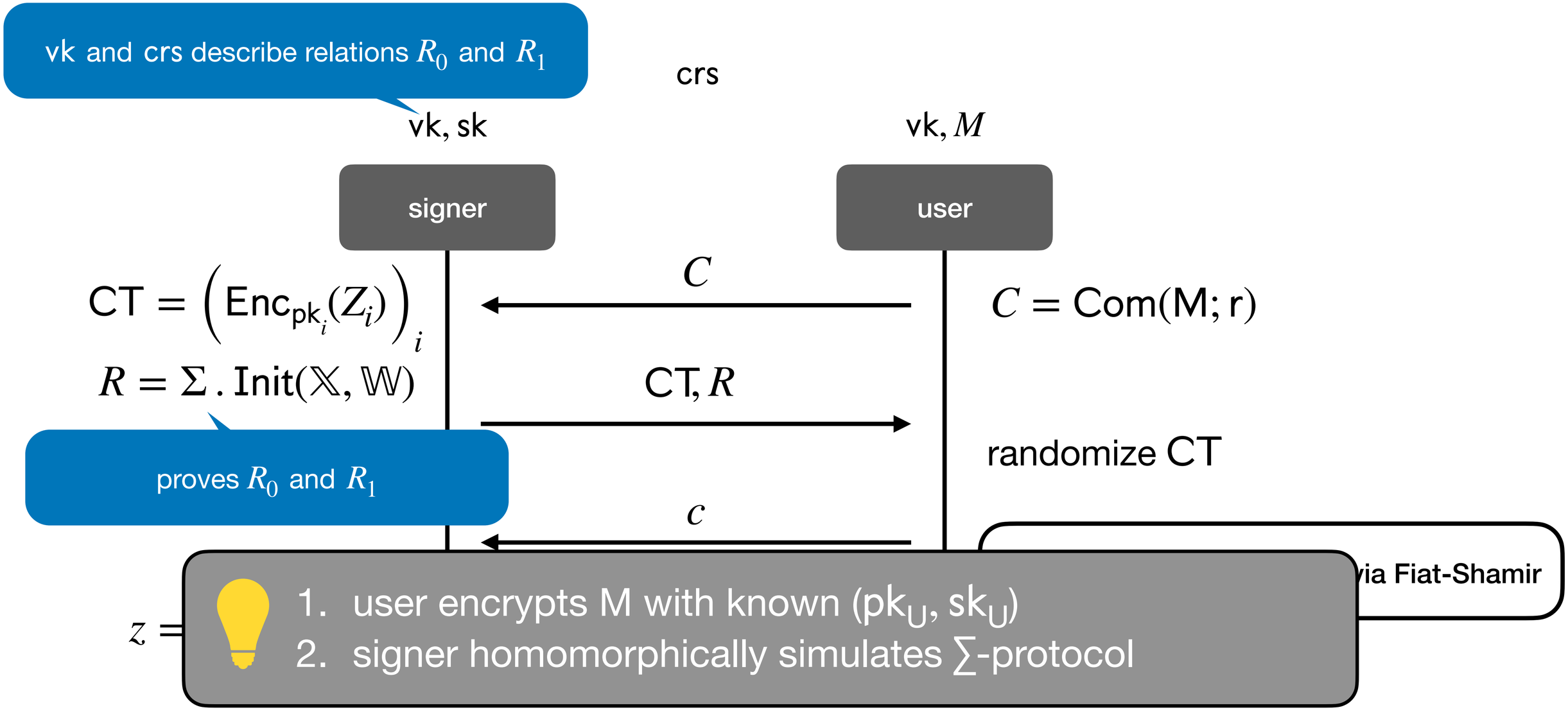

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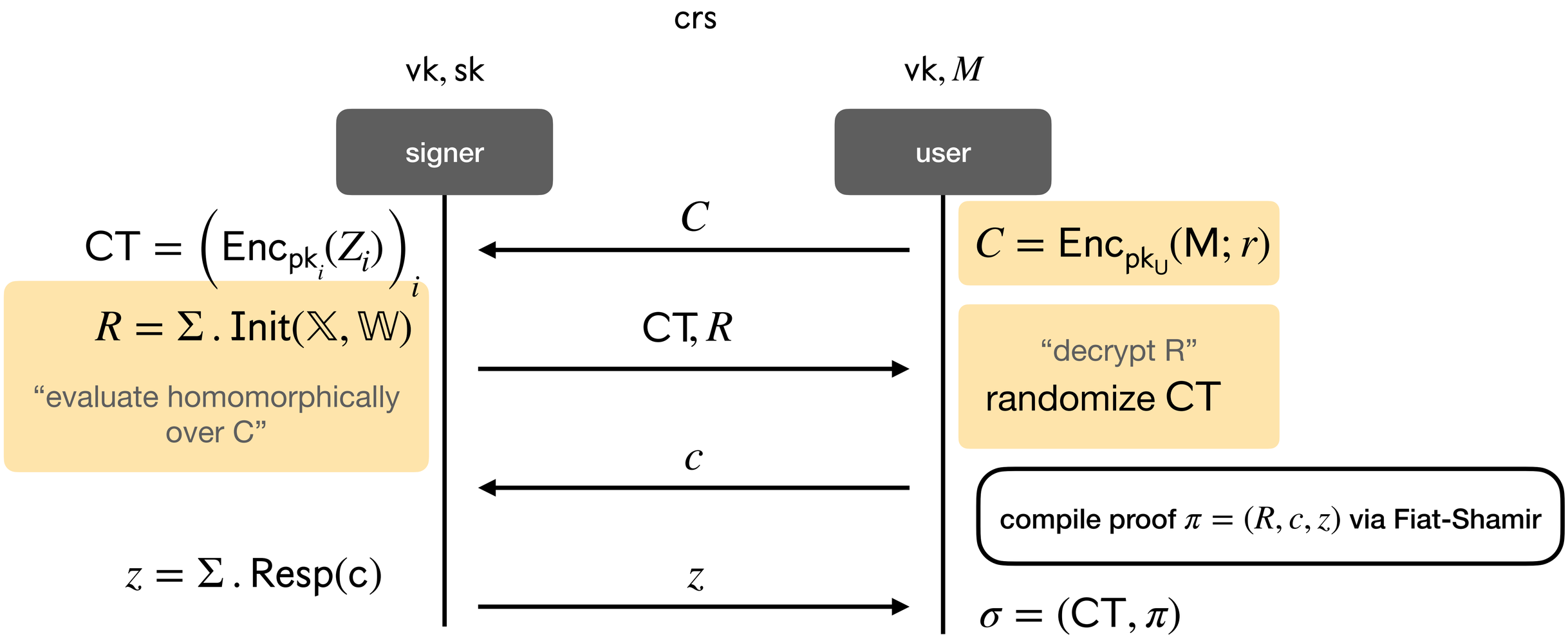

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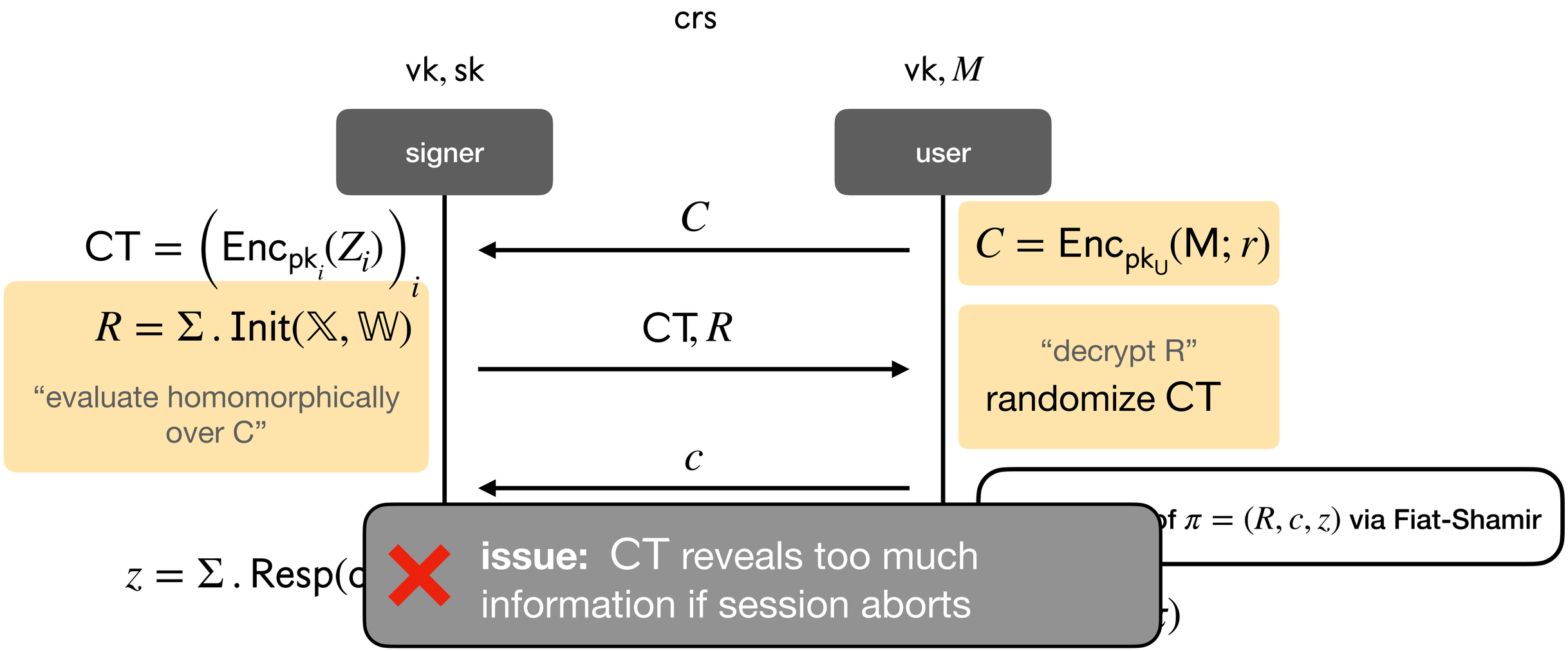

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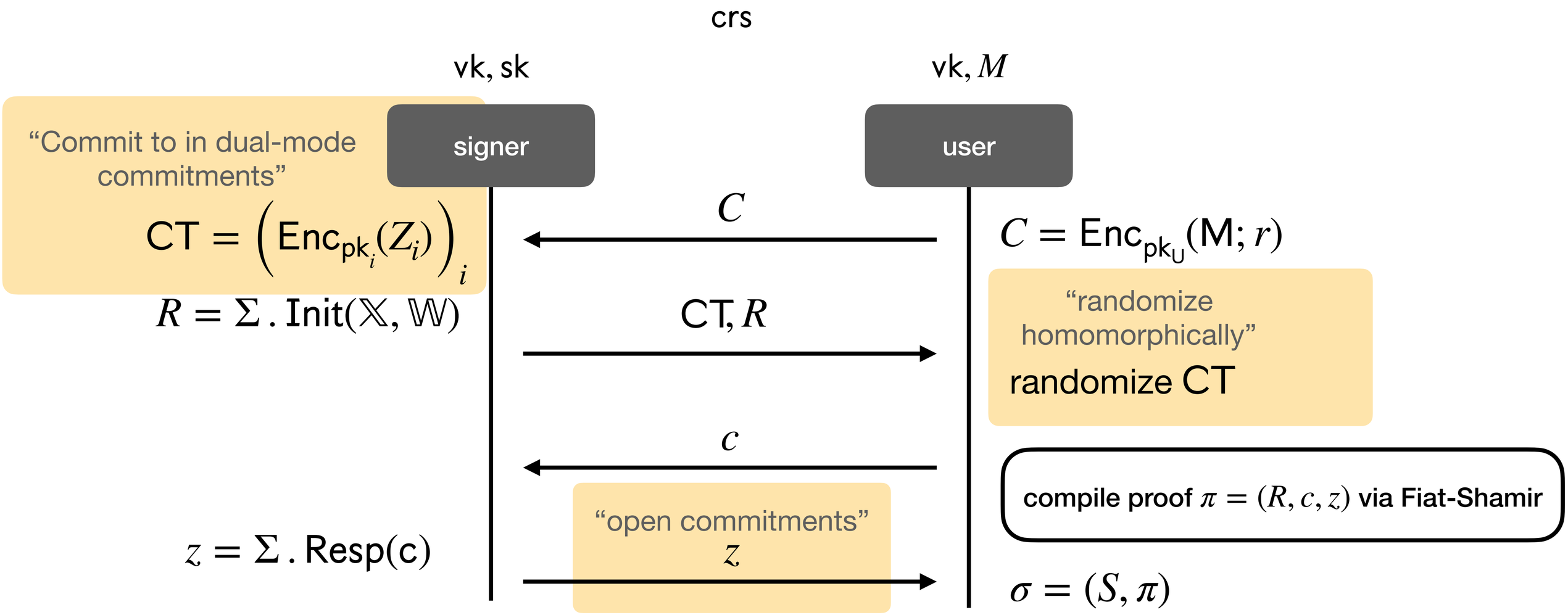

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    → adaptive partitioning

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