Range Proofs

Efficient Range Proofs with Transparent Setup from Bounded Integer Commitments

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» Application

Privacy-Preserving Technologies

» Application

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» Commitments

$$c = \boxed{x\ ;r} \xrightarrow[open]{x,r} \text{Verify}(c, x, r) = 1$$

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- **Hiding**: The commitment does not reveal x.
- **Binding**: The commitment *can* not be opened to something else than x.

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* **Msg Space**: $x \in \mathbb{Z}_q$

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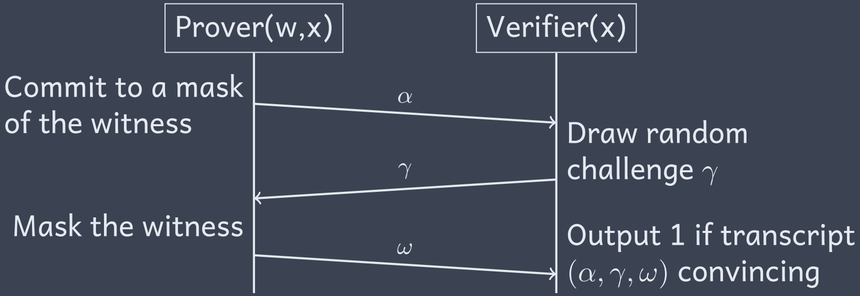
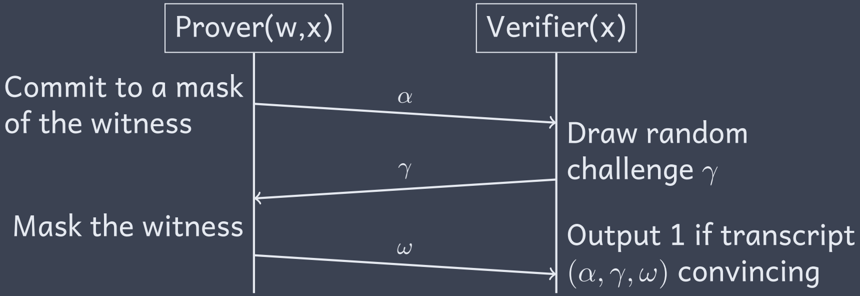
Properties

- **Hiding**: The commitment does not reveal x.
- **Binding**: The commitment can not be opened to something else than x.
- **Msg Space**: $x \in \mathbb{Z}_q$
- **Homomorphy**:
  - Additive: $\boxed{x_0\,;r_0} + \boxed{x_1\,;r_1} = \boxed{x_0 + x_1\,;r_0 + r_1}$
  - Scalar: $n \cdot \boxed{x\,;r} = \boxed{n \cdot x\,;n \cdot r}$

» Σ-Protocols


Prover(w,x)
Verifier(x)
Commit to a mask of the witness
α
Draw random challenge γ
γ
Mask the witness
ω
Output 1 if transcript (α, γ, ω) convincing

» Σ-Protocols



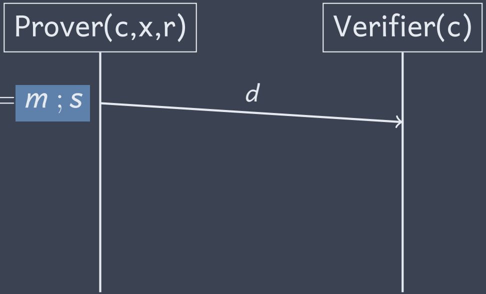
Properties

- **Zero-Knowledge:** Transcripts can be simulated without w.
- **Soundness:** A witness w can be extracted from accepted transcripts.

» Proof of Opening

Protocol

$c = x \,;\, r,\ d = m \,;\, s$



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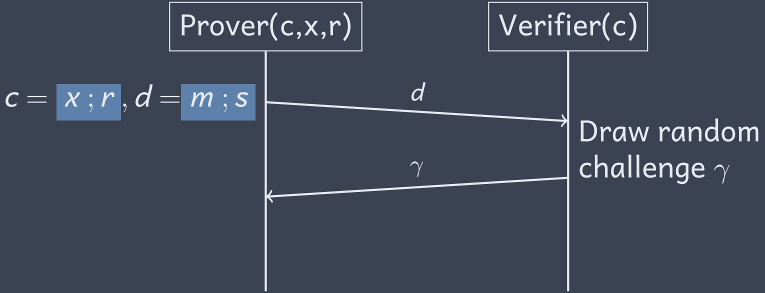

» Proof of Opening

Prover(c,x,r)		Verifier(c)
$c = \boxed{x\,;r}, d = \boxed{m\,;s}$	$\xrightarrow{d}$	
	$\xleftarrow{\gamma}$	Draw random challenge γ
$z = m + \gamma x,$ $t = s + \gamma r$	$\xrightarrow{z,\, t}$	

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$z = m + \gamma x,$ $t = s + \gamma r$	$\xrightarrow{z,\,t}$	$\boxed{m\,;s} + \gamma \boxed{x\,;r} = \boxed{z\,;t}$?

» Proof of Opening

Reminder

- Statement: $c = \boxed{x\,;r}$
- Honest transcript: $d = \boxed{m\,;s}, \gamma, z = m + \gamma x, t = s + \gamma r$

» Proof of Opening

Extraction

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* Statement: $c = \boxed{x\,;r}$
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Assume $\mathcal{A}$ can output accepting transcripts

$$tr_0 = [d, \gamma_0, z_0, t_0] \text{ and } tr_1 = [d, \gamma_1, z_1, t_1]$$

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Then

$$d + \gamma_0 c = \boxed{z_0\ ;t_0} \wedge d + \gamma_1 c = \boxed{z_1\ ;t_1}$$

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$$d + \gamma_0 c = \boxed{z_0\ ;t_0} \wedge d + \gamma_1 c = \boxed{z_1\ ;t_1}$$
$$\Rightarrow (\gamma_0 - \gamma_1)c = \boxed{z_0 - z_1\ ;t_0 - t_1}$$

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$$\Rightarrow (\gamma_0 - \gamma_1)c = \boxed{z_0 - z_1\ ;t_0 - t_1}$$
$$\Rightarrow c = \boxed{(z_0 - z_1)/(\gamma_0 - \gamma_1)\ ;(t_0 - t_1)/(\gamma_0 - \gamma_1)}$$

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Then

$$d + \gamma_0 c = \boxed{z_0 \,;\, t_0} \wedge d + \gamma_1 c = \boxed{z_1 \,;\, t_1}$$
$$\Rightarrow (\gamma_0 - \gamma_1)c = \boxed{z_0 - z_1 \,;\, t_0 - t_1}$$
$$\Rightarrow c = \boxed{(z_0 - z_1)/(\gamma_0 - \gamma_1) \,;\, (t_0 - t_1)/(\gamma_0 - \gamma_1)}$$

In total: $x = (z_0 - z_1)/(\gamma_0 - \gamma_1)$ in $\mathbb{Z}_q$

» Range Proofs

Definition

Zero-knowledge proof for $\mathsf{R} = \{((x, r), (\boxed{x\,;r}, a, b)) \mid x \in [a, b]\}$

» Range Proofs

Approach

Zero-knowledge proof for $\mathrm{R} = \{((x, r), (\boxed{x\,; r}, a, b)) \mid x \in [a, b]\}$

$$x \in [0, 2^\ell) \iff x = \sum_{i=0..\ell-1} x_i 2^i \text{ and } x_i \in \{0, 1\}$$

Approaches

- **Binary Decomposition:**
  - commit to the decomposition
  - prove that $x_i \in \{0, 1\}$
  - most common approach (Lattice, DLOG, ..)

» Range Proofs

Approach

Zero-knowledge proof for $\mathrm{R} = \{((x, r), (x\,;r, a, b)) \mid x \in [a, b]\}$

$$x \in [a, b] \iff x - a, b - x \geq 0$$

Approaches

* **Integer Commitments:**
  * prove that $(b - x)(x - a) = \sum_{i=1..4} x_i^2$
  * $x \in \mathbb{Z}$
  * require trusted setup, large parameters

» Range Proofs

Decomposition

Simplification for $B = b - a$

$$x \in [a, b] \iff x - a \in [0, b - a] \iff x(B - x) = \sum_{i=1..4} x_i^2$$

» Range Proofs

Simplification for $B = b - a$

$$x \in [a, b] \iff x - a \in [0, b - a] \iff x(B - x) = \sum_{i=1..4} x_i^2$$

Optimization [Gro05]

$$x \in [0, B] \iff 1 + 4x(B\text{-}x) = \sum_{i=1..3} x_i^2$$

» Setting

Range Proof

* (generic) commitment: $c_0 = \boxed{x_0 \mod \boldsymbol{q} \,;\, r_0}$
* avoid trusted setup
* optimize efficiency

» Approach I

Idea

Use 3 square decomposition in $\mathbb{Z}_q$:

$$1 + 4x_0(B - x_0) = \sum_{i=1..3} x_i^2$$

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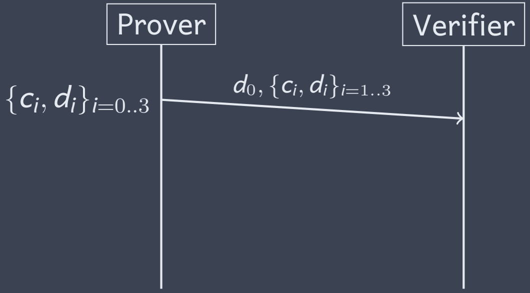

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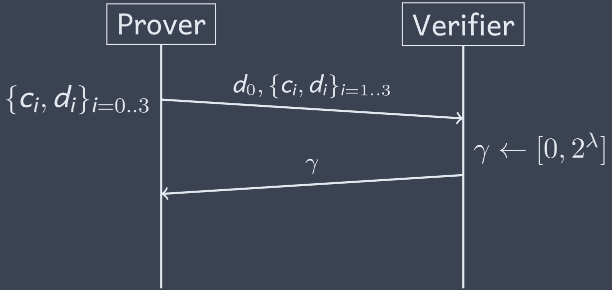

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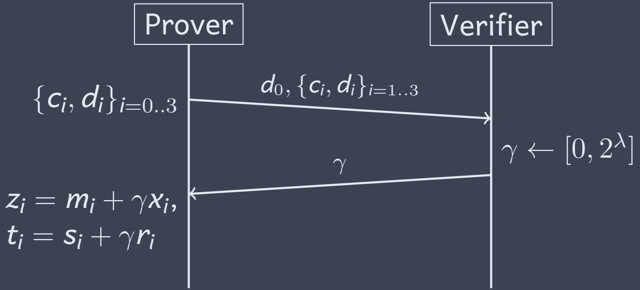

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Prover		Verifier
$\{c_i, d_i\}_{i=0..3}$	$\xrightarrow{d_0, \{c_i, d_i\}_{i=1..3}}$	
	$\xleftarrow{\gamma}$	$\gamma \leftarrow [0, 2^\lambda]$
$z_i = m_i + \gamma x_i$, $t_i = s_i + \gamma r_i$	$\xrightarrow{\{z_i, t_i\}_{i=0..3}}$	

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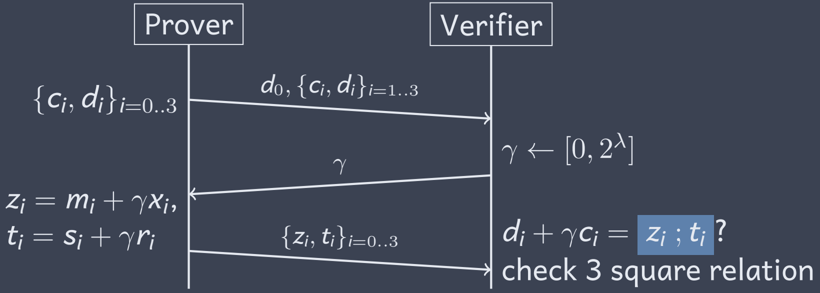
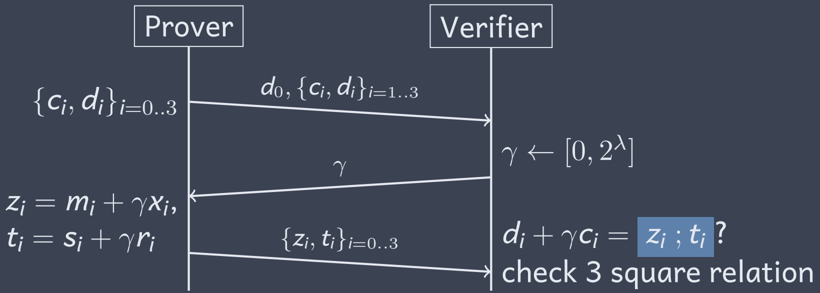


» Approach I

Problem

3 square decomposition in $\mathbb{Z}_q$ does not imply positivity



» Approach II

Idea

Avoid overflows by ensuring short witnesses

Prover		Verifier
$\{c_i, d_i\}_{i=0..3}$	$\xrightarrow{d_0, \{c_i, d_i\}_{i=1..3}}$	
	$\xleftarrow{\gamma}$	$\gamma \leftarrow [0, 2^\lambda]$
$z_i = m_i + \gamma x_i$, $t_i = s_i + \gamma r_i$	$\xrightarrow{\{z_i, t_i\}_{i=0..3}}$	$d_i + \gamma c_i = z_i \,;\, t_i$?
		check 3 square relation
		check z_i short

» Approach II

Problem

Extracted $x_0 = \frac{z_0 - z_0'}{\gamma - \gamma'} \mod q$ not short

Prover | **Verifier**

Prover: $\{c_i, d_i\}_{i=0..3}$

Prover → Verifier: $d_0, \{c_i, d_i\}_{i=1..3}$

Verifier: $\gamma \leftarrow [0, 2^\lambda]$

Verifier → Prover: γ

Prover: $z_i = m_i + \gamma x_i$, $t_i = s_i + \gamma r_i$

Prover → Verifier: $\{z_i, t_i\}_{i=0..3}$

Verifier:
$d_i + \gamma c_i = \boxed{z_i \,;\, t_i}$?
check 3 square relation
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» Approach II

Example

Problem

$$\frac{1}{2} = 3057 \mod 6113 \text{ is large}$$

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Idea

Map fractions in $\mathbb{Z}_q$ to integers via division in $\mathbb{Q}$

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Encoding

$$\left\lfloor \frac{1}{2} \right\rceil = 1 \text{ is small}$$

» Approach III

Relax commitment scheme:

$$\boxed{z \cdot \gamma^{-1} \mod q} \text{ commits to } x = \left\lfloor \frac{z}{\gamma} \right\rceil \in \mathbb{Z}$$

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- binding if z, γ short

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* retains (restricted) homomorphic properties

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Properties

- binding if z, γ short
- retains (restricted) homomorphic properties
- retains shortness
- honest commitment unchanged

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Properties

* binding if z, γ short
* retains (restricted) homomorphic properties
* retains shortness
* honest commitment unchanged

$\rightarrow$ Bounded integer commitment scheme

» Binding Property

Relaxed commitment scheme:

$$z \cdot \gamma^{-1} \mod q \text{ commits to } x = \left\lfloor \frac{z}{\gamma} \right\rceil \in \mathbb{Z}$$

Binding Proof

Receive $z_0 \cdot \gamma_0^{-1} \mod q$ and $z_1 \cdot \gamma_1^{-1} \mod q$

» Binding Property

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* Binding $\Rightarrow z_0 \cdot \gamma_0^{-1} = z_1 \cdot \gamma_1^{-1} \mod q$

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  $\Rightarrow z_0 \cdot \gamma_1 = z_1 \cdot \gamma_0 \mod q$
* Shortness $\Rightarrow z_0\gamma_1 = z_1\gamma_0$ over $\mathbb{Q}$

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  $\Rightarrow \frac{z_0}{\gamma_0} = \frac{z_1}{\gamma_1}$ over $\mathbb{Q}$
  $\Rightarrow \left\lfloor \frac{z_0}{\gamma_0} \right\rceil = \left\lfloor \frac{z_1}{\gamma_1} \right\rceil$ over $\mathbb{Q}$

» Approach III

Obtain range proof for relaxed committed value

Prover | Verifier

$\{c_i, d_i\}_{i=0..3}$ $\xrightarrow{d_0, \{c_i, d_i\}_{i=1..3}}$

$\gamma \leftarrow [0, 2^\lambda]$

$\xleftarrow{\gamma}$

$z_i = m_i + \gamma x_i,$

$t_i = s_i + \gamma r_i$ $\xrightarrow{\{z_i, t_i\}_{i=0..3}}$

$d_i + \gamma c_i = \boxed{z_i \,;\, t_i}$?

check 3 square relation

check z_i short

Extraction

$$\frac{z - z'}{\gamma - \gamma'} \in \mathbb{Z}_q \mapsto \left\lfloor \frac{z - z'}{\gamma - \gamma'} \right\rceil \in \mathbb{Z} \text{ short}$$

» Showing the Decomposition

Requires sending additional group elements and integers.

* Commitments c_1, c_2, c_3 to decomposition x_1, x_2, x_3
* Proof of openings of c_1, c_2, c_3
* Additional mask (DLOG, Lattice, Class Groups)
* Additional commitments (Lattice)

» Limitations - Homomorphism

$\boxed{z \cdot \gamma^{-1} \; ; r}$ commits to $x = \lfloor z/\gamma \rceil \in \mathbb{Z}$

* **Honest:** $\boxed{x_0 \; ; r} + \boxed{x_1 \; ; s} = \boxed{x_0 + x_1 \; ; r + s}$

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* **Honest:** $\boxed{x_0 ; r} + \boxed{x_1 ; s} = \boxed{x_0 + x_1 ; r + s}$
* **Small Constants:**
    * $\boxed{z \cdot \gamma^{-1} ; r} + \boxed{a ; 0} = \boxed{(z + \gamma a) \cdot \gamma^{-1} ; r}$
    * commits to $x + a = \lfloor z/\gamma \rceil + a$

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  * commits to $x + a = \lfloor z/\gamma \rceil + a$
* **Dishonest:**
  * $\boxed{z_0 \cdot \gamma^{-1} \,;\, r} + \boxed{z_1 \cdot \gamma^{-1} \,;\, s} = \boxed{(z_0 + z_1) \cdot \gamma^{-1} \,;\, r + s}$
  * commits to $\lfloor z_0/\gamma \rceil + \lfloor z_1/\gamma \rceil + \{0, 1\}$
  * worse for non-equal denominator

» Limitations - Homomorphism

$\boxed{z \cdot \gamma^{-1}\,;r}$ commits to $x = \lfloor z/\gamma \rceil \in \mathbb{Z}$

- Honest: $\boxed{x_0\,;r} + \boxed{x_1\,;s} = \boxed{x_0 + x_1\,;r+s}$
- Small Constants:
  - $\boxed{z \cdot \gamma^{-1}\,;r} + \boxed{a\,;0} = \boxed{(z+\gamma a) \cdot \gamma^{-1}\,;r}$
  - commits to $x + a = \lfloor z/\gamma \rceil + a$
- Dishonest:
  - $\boxed{z_0 \cdot \gamma^{-1}\,;r} + \boxed{z_1 \cdot \gamma^{-1}\,;s} = \boxed{(z_0 + z_1) \cdot \gamma^{-1}\,;r+s}$
  - commits to $\lfloor z_0/\gamma \rceil + \lfloor z_1/\gamma \rceil + \{0, 1\}$
  - worse for non-equal denominator

$\rightarrow$ ensure that committed integers are small enough

$\rightarrow$ be careful about guarantees

» Limitations - Group Size

Need to ensure no overflow in square decomposition:

$$1 + 4x_0(B - x_0) = \sum_{i=1..3} x_i^2$$

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$\rightarrow$ ensure that both sides are smaller than the modulus q
$\rightarrow$ leads to large group size

» Optimizations

$$z_i = m_i + \gamma x_i$$

* **Rejection Sampling:** shorter masks $\rightarrow$ smaller modulus

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* **Rejection Sampling**: shorter masks $\rightarrow$ smaller modulus
* **Repetitions**: shorter challenge $\rightarrow$ smaller modulus
* **Fiat-Shamir**: non-interactive range proof

» Settings

* **DLOG**: improves on Bulletproofs [BBB+18]
* **Lattice**: efficient for large batches
* **Class Groups**: first concretely efficient unbounded integer commitment scheme without trusted setup

» DLOG

Pedersen Commitments

- $\mathbb{G}$: group with prime order q
- $g, h \in \mathbb{G}$: generators
- $x \in Z_q, r \leftarrow [0, 2^{2\lambda}]$

$$\boxed{x\,;r} = g^x h^r$$

- based on DLSE assumption

» DLOG

Pedersen Commitments

* $\mathbb{G}$: group with prime order q
* $g, h \in \mathbb{G}$: generators
* $x \in Z_q, r \leftarrow [0, 2^{2\lambda}]$

$$\boxed{x\,;r} = g^x h^r$$

* based on DLSE assumption

* Decomposition: use (honest) homomorphic properties
* Efficient range proofs for single x

» DLOG

Security Parameter	80	128
Range	$B = 32$	
Proof size	88%	81%
Prover's work	12%	11%
Range	$B = 64$	
Proof size	89%	80%
Prover's work	6%	6%

Our work compared to Bulletproofs [BBB+18]. Prover's work compared in group multiplications.

» Lattices

[BDL+18] commitments

* $q \in \mathbb{N}$ prime
* matrix $A \in \mathbb{Z}_q^{(l1+n)\times(l1+n+l2)}$
* $\vec{x} \in \mathbb{Z}_q^n, \vec{r} \leftarrow D_\sigma^{l1+n+l2}$

$$\boxed{\vec{x}\,;\vec{r}} = A \cdot \vec{r} + (\vec{0} \,\|\, \vec{x})$$

* based on SIS and LWE assumption

* Decomposition with polynomial trick
* Perform range proof for each component
* Amortized proofs more efficient than the state of the art in standard lattice setting

» Class Groups

Pedersen Commitments

* Groups $\mathbb{G}$ with hidden order
* based on ORD and SI assumption
* extraction differs:

$$x = \frac{z}{2^\ell}$$

» Class Groups

Pedersen Commitments

* Groups $\mathbb{G}$ with hidden order
* based on ORD and SI assumption
* extraction differs:

$$x = \frac{z}{2^\ell}$$

* Same structure as DLOG version
* Larger group elements
* No bounds on the committed values

» References

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